Alfvén current drive in magnetic traps

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General approach to the Alfvén current-drive problem is developed in this paper. The covariant-form expression for the longitudinal drag force, which can be applied to any magnetic traps (both closed and open), is obtained. For closed magnetic traps, the surface-averaged high-frequency driving force is derived. For axially symmetric tokamaks with an arbitrary transverse cross section, a simple expression for the force is found. It is shown that the magnetohydrodynamic approach can be used to get the oscillating currents on which the time-averaged force depends.

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The problem of noninductive current drive in cylindrical plasma and in circular cross-section tokamaks is already very well understood [1–7]. Since the beginning of the investigations on this problem, it has been clear that there are significant difficulties in using the current drive in a tokamak reactor. Thus, in the case of the lower-hybrid current drive the efficiency of this process drops as the plasma density increases. For the Alfvén waves, there is also an opinion that the efficiency drops as a result of wave absorption by the trapped particles.

Ohkawa proposed [3] that the current in a magnetized plasma can be maintained by means of forces, depending on the high-frequency (hf) field amplitude gradients, and his idea was developed in [4–7]. Some new hopes then appeared, connected with the possibility to increase the current-drive efficiency. It was shown, for the cylinder plasma case [2], that the local efficiency of Alfvén wave current drive can be increased by one order due to gradient forces, e.g., for kinetic Alfvén waves and global Alfvén waves at some range of the phase velocity. For tokamaks, this additional nonresonant current drive does not depend on the trapped particle effects. As supposed [1,2], trapped particles reduce strongly the Alfvén current-drive efficiency in tokamaks [2].

In this paper, an attempt is made to clarify some general aspects of this problem for arbitrary magnetic traps. To derive general expressions for current-drive forces, we proceed from the time-averaged motion equation [7,8]

$$\begin{aligned} M_\alpha \frac{\partial}{\partial t}(N_{\alpha 0}\mathbf{U}_{\alpha 0}) = & -M_\alpha \nabla_i (N_{\alpha 0} U^i_{\alpha 0}\mathbf{U}_{\alpha 0}) - \boldsymbol{\nabla} P_{\perp\alpha 0} \\ & -\nabla_i [h^i_0 \mathbf{h}_0 (P_{\parallel\alpha 0} - P_{\perp\alpha 0})] \\ & + e_\alpha N_{\alpha 0}\mathbf{E}_0 + \frac{e_\alpha N_{\alpha 0}}{c}[\mathbf{U}_{\alpha 0}\times \mathbf{B}_0] \\ & + \mathbf{R}_{\alpha 0} + \mathbf{F}_\alpha . \end{aligned} \qquad (1)$$

Here the time-averaged current-drive general force $\mathbf{F}_\alpha$ is equal to

$$\mathbf{F}_\alpha = \mathbf{F}_{n\alpha} + \mathbf{F}_{d\alpha} + \mathbf{F}_{p\alpha}, \qquad (2)$$

where

$$\mathbf{F}_{n\alpha} = e_\alpha (N_{\alpha\omega}\mathbf{E}_\omega)_0,$$

$$\mathbf{F}_{d\alpha} = \frac{1}{c}([\mathbf{j}_{\alpha\omega}\times \mathbf{B}_\omega])_0,$$

$$\mathbf{F}_{p\alpha} = -4\pi \nabla_i \left(\frac{j^i_{\alpha\omega}\mathbf{j}_{\alpha\omega}}{\omega^2_{p\alpha}} \right)_0 .$$

The subscripts 0 and ω denote the time-averaged and the hf values, respectively. Here we define

$$N_{\alpha 0}\mathbf{U}_{\alpha 0} = (N_\alpha \mathbf{V}_\alpha)_0 . \qquad (3)$$

The term $\mathbf{F}_{p\alpha}$ contradicts the results of the kinetic approach [2,7,9]. This problem requires special consideration. A possible explanation of this effect is a collisional smoothing of the plasma pressure over the wave field in the hydrodynamic approach.

We are interested in a longitudinal component of the hf force, averaged along the magnetic field line (see, e.g., [10])

$$\overline{F}_{\alpha\parallel} = \langle F_{\alpha\parallel} B_0 \rangle / B_s, \qquad (4)$$

where

$$\langle \cdots \rangle = \oint (\cdots) \frac{dl}{B_0} \Big/ \oint \frac{dl}{B_0}. \qquad (5)$$

The surface-averaged Ohm law is Eq. (1) (see, e.g., [10])

$$\langle J_\parallel B_0 \rangle = \sigma_\parallel \left(\langle E_{0\parallel} B_0 \rangle - \frac{1}{e_e N_e} \langle \mathbf{B}_0 \cdot (\boldsymbol{\nabla} \cdot \hat{\boldsymbol{\pi}}_{\parallel e}) \rangle + \frac{1}{e_e N_e} \langle \mathbf{F}_e \cdot \mathbf{B}_0 \rangle \right). \qquad (6)$$

After some simplifications, using the continuity and Maxwell equations, we can obtain, from Eq. (2), the general expression for the current-drive longitudinal drag force in the covariant form, which can be used for any magnetic traps (both closed and opened) in any coordinate system (the indices ω are omitted below)

$$F_{\alpha\|} = \frac{i}{4\omega}\left\{\frac{h_0^i}{\sqrt{g}}\frac{\partial}{\partial x^k}\left(j_\alpha^{*k}E_i\sqrt{g}\right) + E_i\left(h_0^k\frac{\partial}{\partial x^k}\right)j_\alpha^{*i} - \left(h_0^k\frac{\partial}{\partial x^k}\right)(\mathbf{j}_\alpha^*\cdot\mathbf{E}) - \text{c.c.}\right\} - \pi\mathbf{h}_0\nabla_k\frac{(j_\alpha^{*k}\mathbf{j}_\alpha + \text{c.c.})}{\omega_{p\alpha}^2}. \qquad (7)$$

After the surface averaging of Eq. (5), for closed magnetic traps, we get

$$\langle F_{\alpha\|}B_0\rangle = \left\langle\frac{iB_0}{4\omega}\left\{E_\|\left(j_\alpha^{*i}\frac{\partial}{\partial x^i}\right)\ln\sqrt{g} + h_0^i\frac{\partial}{\partial r}\left(j_\alpha^{*r}E_i\right) + E_i\left(h_0^k\frac{\partial}{\partial x^k}\right)j_\alpha^{*i} - \text{c.c.}\right\} - \pi\mathbf{B}_0\nabla_k\frac{(j_\alpha^{*k}\mathbf{j}_\alpha + \text{c.c.})}{\omega_{p\alpha}^2}\right\rangle. \qquad (8)$$

Here the radial coordinate r designates any of the magnetic field surface functions, e.g., the toroidal ϕ or poloidal χ magnetic field fluxes or the plasma volume V. The poloidal θ and toroidal ζ angle coordinates are supposed to be chosen so that the magnetic field lines are straight in these coordinates. The operator ∇_k should be used in accordance with the covariant differentiation rules

$$\nabla_k j^i = \frac{\partial j^i}{\partial x^k} + \Gamma_{km}^i j^m, \qquad (9)$$

where Γ_{km}^i are the Cristoffel symbols and g is the metric tensor determinant. In the Hamada coordinates [11], the first term on the right-hand side of Eq. (7) is equal to zero.

For axially symmetric tokamaks with an arbitrary cross section, we are especially interested in the terms that will be derived after surface averaging simply to the next

$$\frac{i}{4\omega}\left\langle B_0\left\{h_0^i\frac{\partial}{\partial r}\left(j_\alpha^{*r}E_i\right) + E_i\left(h_0^k\frac{\partial}{\partial x^k}\right)j_\alpha^{*i} - \text{c.c.}\right\}\right\rangle = \frac{i}{4\omega}\int_0^{2\pi}d\theta\left\{\frac{\partial}{\partial r}\left(j_\alpha^{*r}E_\theta\right) + q\frac{\partial}{\partial r}\left(j_\alpha^{*r}E_\zeta\right) + E_i\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta}\right)j_\alpha^{*i} - \text{c.c.}\right\}\Big/\oint\frac{dl}{B}. \qquad (10)$$

The safety factor q is equal to $q = \phi'/\chi'$. Here the prime denotes the radial derivative.

Note that there is no expression such as this in Ref. [12]. We suppose that there the practically single particle approach was used, having neglected the collision frequency ν_e, and the necessary term $\nu_e(N_\omega j_{\|\omega})_0$ and as a result the terms $(N_\omega\mathbf{E}_\omega)_0$ and partially $(\mathbf{j}_\omega\times\mathbf{B}_\omega)_0$ had been lost.

It can be seen from Eq. (8) that to derive the time- and surface-averaged longitudinal current-drive force, we need to find the oscillating current expressions. Here we use the magnetohydrodynamic approach to calculate currents [13,14], which is valid for all the collisional regimes. Supposing the electric field and others macroscopic values of the plasma to oscillate with the frequency ω as $\exp(-i\omega t)$, we get

$$\mathbf{j}_{\alpha\perp} = \frac{e_\alpha}{M_\alpha\left(\omega_{c\alpha}^2 - \omega^2\right)}\left(\omega_{c\alpha}[\mathbf{h}\times\mathbf{A}_\alpha] + i\omega\mathbf{A}_\alpha\right), \qquad (11)$$

where

$$\mathbf{V}_{0\alpha} = (c/e_\alpha N_\alpha B_0)[\mathbf{h}\times\nabla p_{0\alpha}],$$

$$\mathbf{A}_\alpha = -e_\alpha N_\alpha\mathbf{E}_\perp - \frac{e_\alpha N_\alpha}{c}[\mathbf{V}_{0\alpha}\times\mathbf{B}] + \nabla_\perp p_{\perp\alpha} + (p_{\|\alpha} - p_{\perp\alpha})\nabla_\perp\ln B.$$

We used the definitions of p_α and $\pi_{\|\alpha}$ given by Braginskii [13]:

$$p_\alpha = \frac{1}{3}M_\alpha\int v^2 f_\alpha d\mathbf{v},$$

$$\pi_{\|\alpha} = M_\alpha\int\left(v_\|^2 - \frac{1}{3}v^2\right)f_\alpha d\mathbf{v}. \qquad (12)$$

Here f_α is the distribution function of ions or electrons, which is determined from the kinetic equation (see, e.g., [15]), and hence the Landau damping information is enclosed in these terms. We have taken into account only the longitudinal component of the viscosity, as usual, which is important for a fully ionized, weakly collisional plasma in different situations [10]. The viscosity equation is [16]

$$\nabla\cdot\hat{\pi} = \frac{3}{2}\left\{[\mathbf{h}(\nabla\cdot\mathbf{h}) + (\mathbf{h}\cdot\nabla)\mathbf{h}]\pi_\| + \mathbf{h}(\mathbf{h}\cdot\nabla)\pi_\|\right\} - \frac{1}{2}\nabla\pi_\|. \qquad (13)$$

We used also the definitions

$$p_\perp = p - \frac{1}{2}\pi_\|, \qquad p_\| = p + \pi_\|. \qquad (14)$$

Thus, to derive currents in Eq. (8),

$$\mathbf{j} = j_\|\mathbf{h} + \mathbf{j}_\perp\ , \qquad (15)$$

we need to calculate only the three scalar values $p_\perp, p_\|$ [Eqs. (12) and (14)], and $j_\|$

$$j_{\alpha\|} = e_\alpha\int d\mathbf{v}v_\| f_\alpha. \qquad (16)$$

For example, we have the expression for $p_\perp$ in the cylindrical plasma case

$$p_{\perp\alpha} = \frac{ie_\alpha N_{0\alpha}v_{T\alpha}^2}{\omega\omega_{c\alpha}}\left(-i\hat{M}_{b\alpha}E_r + \hat{M}_{r\alpha}E_b + \hat{M}_{\|\alpha}E_\|\right). \qquad (17)$$

Here the denotations are

$$\hat{M}_{r\alpha} = 2\left(\Lambda_\alpha - 1\right)\left(\frac{\partial}{\partial r} + \frac{1}{r}\right) - \chi_N - \chi_T,$$

$$\Lambda_\alpha = 1 + i\sqrt{\pi}Z_\alpha W(Z_\alpha), \quad \hat{M}_{b\alpha} = 2k_b\left(\Lambda_\alpha - 1\right),$$

$$W(x) = \exp(-x^2)\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^x \exp(t^2)dt\right),$$

$$Z_\alpha = \frac{\omega}{k_\| v_{T\alpha}},$$

$$\hat{M}_{\|\alpha} = -\frac{\omega_{c\alpha}\omega\Lambda_\alpha}{k_\| v_{T\alpha}^2} + \frac{k_b\chi_n}{k_\|}\Lambda_\alpha + \frac{k_b\chi_T}{2k_\|}\left[1 + \Lambda_\alpha\left(1 + 2Z_\alpha^2\right)\right],$$

$$\chi_A = \partial \ln A/\partial r.$$

For the waves with frequency ω under the condition $\omega \ll \omega_{ce}$, we obtain from Eq. (11), in the zero approximation of the ratio ω/ω_{ce},

$$j_e^r = \frac{e_e}{M_e\omega_{ce}}\left\{e_e N_e E_b - i\hat{k}_b p_{\perp e} + h_{0\zeta}(p_{\|e} - p_{\perp e})\frac{\partial}{\partial\theta}\ln B_0\right\}. \quad (18)$$

Here the denotations are

$$E_b = [\mathbf{E}\times\mathbf{h}]^r = \frac{1}{\sqrt{g}}\left(E_\theta h_{0\zeta} - E_\zeta h_{0\theta}\right),$$

$$i\hat{k}_b = -[\mathbf{h}\times\nabla]^r = \frac{1}{\sqrt{g}}\left(h_{0\zeta}\frac{\partial}{\partial\theta} - h_{0\theta}\frac{\partial}{\partial\zeta}\right),$$

where $h_{0\zeta} = g_{33}h_0^\zeta$ and $h_{0\theta} = g_{22}h_0^\theta$. Substituting these expressions into Eqs. (8) and (10), we get almost the same form of expression [with the exception of the last term in Eq. (18)] for the longitudinal force as in the cylindrical case [2,7]. For the concrete systems, it is necessary to derive the connection between the oscillating current components and the electric field components and to find the radial dependence of the oscillating electric field. For the cylindrical case we obtain the coincidence of the Alfvén current-drive calculations by means of the magnetohydrodynamic approach and the direct estimation of the oscillating currents on the base of a kinetic equation.

In conclusion, we have developed in this paper a general approach to the current-drive problem for closed magnetic traps. The covariant-form expression for the longitudinal drag force, which can be applied to any magnetic traps (both closed and opened), is obtained. For closed magnetic traps, the surface-averaged drag force is derived. For axially symmetric tokamaks with an arbitrary transverse cross section, the simple expression for the parts of the force, connected with the absorbed power and the radial gradient of the electric fields amplitudes, is found.

It is shown that this magnetohydrodynamic approach can be used to get the rf currents that the time-averaged force depends on. To find the Landau damping it is necessary to calculate, on the base of the drift kinetic equation, only three scalar values: the hf transverse and longitudinal partial electron pressures and the hf longitudinal current. It is also shown that the gradient part of the drag force, in axially symmetric tokamaks, with an arbitrary transverse cross section, looks like the one in a plasma cylinder.

It is possible to arrive at some conclusion on the efficiency of the current drive by means of the gradient (with the radial derivative) term in Eqs. (8) and (10). It depends only on the Landau damping on the different kinds of particles, but not on the relative amounts of these particles. At the same time, the term with an absorbed power [17] decreases with the increase of the trapped particle amount, as it is usually supposed, and can disappear in the case of standing waves.

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